SOLID STATE PHYSICS 1

- August 2004 -

Do not forget to write your full name and student number on each sheet.
Please use separate sheets for each of the four problems.
Physical constants are on sheet 5.

I. The following questions should be answered very briefly (1-2 sentences); 2 points for each correct response.

a) What is the Van der Waals interaction? (no formulae, only explanation)

b) Define a Wigner Seitz cell; include drawing

c) In a crystal with 3 atoms in the primitive cell, how many branches does the phonon dispersion relation have?

d) What is a Fermi surface?

e) What does the excitation of a magnon correspond to?

f) Apply Hund's rule to find the angular momentum quantum number, the spin quantum number and the total angular + spin momentum quantum number for Pr^{3+} in the configuration $4f^2 5s^2 p^6$. Give also the spectroscopic notation of the ground state.

g) How can you experimentally determine the band gap of a given semiconductor?

h) When Nb is cooled below 9,5 K the conduction electrons go from a disordered to an ordered state. What is this ordered state called? What is the nature of the ordering? What happens to the resistivity of Nb?

i) Sketch the phase transitions in a centrosymmetric antiferroelectric crystal below and above T_c and when an electric field is applied for $T<T_c$. Make a sketch for a 2D case.

j) What are the common point imperfections in crystals? Name two crystal properties which are influenced by them.

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II. (a) Explain the difference between the Fermi velocity and the drift velocity of free electrons in solids.

(b) The density of states of electrons according to the free electron model is given by

$$D(E) = D_0 E^{1/2}$$

where E is the electron energy, and $D_0 = (2m)^{1/2} m_e V / (\pi^2 \hbar^3)$ where m_e is the electron mass and V is the volume of the material.

i. Show that the Fermi velocity is given by

$$v_F = \frac{\hbar}{m}\left(3\pi^2 n\right)^{1/3}$$

where n is the free electron density in the material.

ii. Calculate the Fermi velocity for electrons in aluminium if the free electron density is 1.81×10^{29} m^{-3}

(c) The response of a set of free electrons in a material to an external force F in the x direction can be modelled using the following transport equation:

$$\frac{dv_D}{dt} + \frac{v_D}{\tau} = \frac{F}{m}$$

where v_D is the drift velocity in the x direction, t is time, τ is the relaxation time and m is the electron mass.

i. Solve this equation in the steady state for a constant applied electric field to show that the electrical conductivity of the material, according to the model, is given by

$$\sigma = ne^2\tau / m$$

where n is the free electron density and e is the electronic charge.

ii. If the electrical conductivity of aluminium at room temperature is $3.8 \times 10^7\ \Omega^{-1}$m^{-1}, and using the free electron density given above in part b(ii), calculate the relaxation time and drift velocity of a free electron in an electric field of 100 Vm^{-1}.

iii. If the temperature of the metal is raised above room temperature, would you expect the electrical conductivity of aluminium to increase, decrease, or stay the same? Why?

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III. (a) Explain what is meant by the terms *dispersion relation* and *normal mode* in lattice dynamics.

(b) Derive the dispersion relation for a one-dimensional chain of N atoms, equally spaced with separation a, and all with the same mass M. Each atom is coupled to ita nearest neighbours by a spring with force constant K. Sketch the behaviour of ω in the range $-\pi/a \leq \mathbf{k} \leq \pi/a$.

(c) Derive an expression for the group velocity of a mode and comment on the value of the group velocity at $\mathbf{k}\rightarrow 0$ and at the Brillouin zone boundary ($\mathbf{k}=\pi/a$)

(d) The density of states in frequency space for oscillations of the chain is given by

$$G(\omega) = \frac{2N}{\pi}\left(\frac{4K}{M} - \omega^2\right)^{-1/2}.$$

Sketch this function, indicating the maximum allowed frequency. Add in the same graph the density of states i) assumed in the Debye model; ii) assumed in the Einstein model.

(e) (For additional points) Now insert an atom of mass *m* ($\neq$M) between each atom of mass *M*, to create a chain ...mMmMmM... with the distance between atoms equal to *a*/2.
What new feature in the density of states appears as a result of the insertion of the additional atoms? Illustrate your answer with a rough sketch of the new density of states.

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IV. (a) Explain what the two types of extrinsic semiconductors are (use sketches of the density of states for both types).

(b) Explain what happens when a pn junction is formed.

(c) In a particular semiconductor there are 10^{13} donors/ cm^3 with a ionisation energy E_d of 1 meV and an effective mass amounting 1/100 of the electron mass, m.

Estimate the concentration of conduction electrons at 4K.

What is the value of the Hall coefficient? Assume no acceptor atoms are present and that $E_g >> k_B T$.

Planck's constant	$\hbar$	1.055×10^{-34} Js
Boltzmann's constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
speed of light	c	3.0×10^{8} m s^{-1}
elementary charge	e	1.60×10^{-19} C
mass of the electron	m	9.11×10^{-31} kg
Bohr magneton	μ_B	9.27x10-24 J T^{-1}

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